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# Joule Heating Effect on Entropy Generation in a Chemically Reacting Nanofluid through Vertical Channel

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**ABSTRACT**
*Joule heating effect on a chemically reacting nanofluid in vertical channel has been investigated in this paper. By using HAM, the non dimensionalized governing equations are solved. Effect of chemical reaction, magnetic parameter and Joule heating on entropy generation, Bejan number, nanoparticle volume fraction, temperature and velocity are investigated and represented geometrically.*
***KEYWORDS;-****Chemical reaction, Entropy generation, HAM, Joule heating, Nanofluid*

## I. INTRODUCTION

Nanofluids, which are conventional heat transfer fluids pioneered by Choi, [1], Nanofluids are combination of nano-sized solid particles, have been the subject of intensive study by the several researchers since it was reported experimentally that nanofluids exhibits high thermal conductivity compared to other heat transfer fluids[1]. Using the Buongiorno method Hang *et al.*[2], examined the mixed convection flow of nanofluids in a vertical channel. Nield and Kuznetsov, [3], discussed the forced convection nanofluid flow in a channel. Das *et al.*[4], investigated the nanoparticle volume fraction of nanofluid in a vertical plates. Joule heating is produced by inter communication among the atomic ions that compose the body of the conductor and the moving charged particles that form the current and Joule heating plays an important role on MHD heat transfer flow. Hayat *et al*. [5], analyzed the forced convection flow of a nanofluid in a channel by considering Joule heating effect along thermal radiation and viscous dissipation taking in to account.

On the other hand, the effect of chemical reaction on nanofluid may play an important role in many material processing systems. These include flow in packed bed electrodes, co-current buoyant upward gas-liquid [6]. Habibis*et al.* [7], study the flow reversal of chemical reacting fluid flow in a channel. Kothandapani*et al.* [8], investigated the effect of chemical reaction on flow of a nanofluid in presence of inclined magnetite field in a vertical channel. Babulal and Dulalpal, [9], study the combination of chemical reaction parameter and Joule heating parameter effects on MHD mixed convective flow of viscous dissipating fluid on a vertical parallel plates. Mridulkumar, [10], analyses the chemically reacting magnetohydrodynamic flow with Joule heating effect over an exponentially stretching sheet. The amount of irreversibility related to the real processes is measure by entropy generation. When entropy generation is consider the quality of energy decreases.Hence, by applying the entropy generation the performance of the system can be improved. Bejan, [11, 12], developed the optimization method of entropy generation and its applications are introduced in science and engineering.

## II. FORMULATION OF PROBLEM

Consider an electrically conducting incompressible nanofluid flow passing through a vertical channel of width *2d*. The temperature maintained at the plate $y=-d$ is $T_1$and nanoparticle volume fraction is $\phi_1$, whereas $T_2$ and $\phi_2$ maintained at the plate $y=d$ respectively. A uniform magnetic field $B_0$ is taken in *y*direction. Further, in buoyancy term the characteristics of fluid are taken as constant apart from the density and the joule heating effects along with chemical reaction are taken into consideration. $u$(y) is the velocity, $T$(y) is temperature and $\phi$(y)is nanoparticle volume fraction respectively. Under these assumptions, Equations of the conservancy of total thermal energy, mass, momentum and nanoparticle concentration with considering the nanofluid model proposed by Buongiorno, [13], are as follows:

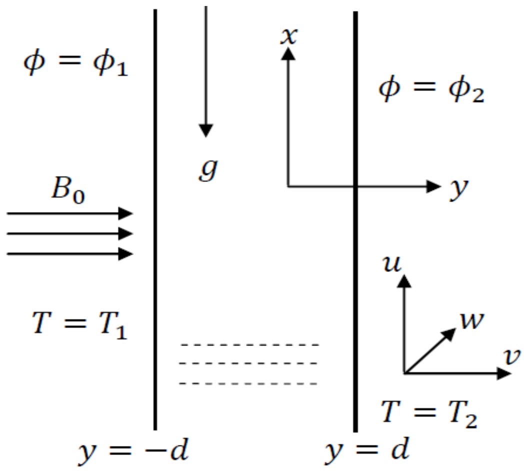



**Fig. 1.** Geometry of the problem

$$\frac{\partial v}{\partial y} = 0 \quad \ldots(1)$$

$$\rho_f v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} + (1-\phi_m)\rho_{f_0} g \beta_T (T - T_1) - (\rho_p - \rho_{f_0}) g (\phi - \phi_1) - \sigma B_0^2 u \quad \ldots(2)$$

$$v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{2\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 + \tau \left[ D_B \frac{\partial T}{\partial y}\frac{\partial \phi}{\partial y} + \frac{D_T}{T_m}\left(\frac{\partial T}{\partial y}\right)^2 \right] + \frac{1}{\rho c_p}\sigma B_0^2 u^2 \quad \ldots(3)$$

$$\frac{\partial \phi}{\partial y} v = \frac{D_T}{T_m}\frac{\partial^2 T}{\partial y^2} + D_B \frac{\partial^2 \phi}{\partial y^2} - k_1(\phi - \phi_1) \quad \ldots(4)$$

It is to be noted from (1) that $v = v_0$ (a constant).The boundary conditions

$$\begin{aligned} u = 0,\quad v = v_0,\quad T = T_1,\quad \phi = \phi_1 \quad \text{on} \quad y = -d, \\ u = 0,\quad v = v_0,\quad T = T_2,\quad \phi = \phi_2 \quad \text{on} \quad y = d, \end{aligned} \quad \ldots(5)$$

Using the below transformations in equations (1) - (5)

$$\eta = \frac{y}{d}, f = \frac{u}{u_0}, p = \frac{\mu u_0}{d^2} P, \theta = \frac{T - T_1}{T_2 - T_1}, S = \frac{\phi - \phi_1}{\phi_2 - \phi_1}, \quad \ldots(6)$$

The resultant nonlinear differential equations are

$$f'' - Rf' + \frac{Gr}{Re}(\theta - Nr\, S) + M^2 f - A = 0 \quad \ldots(7)$$

$$\theta'' - R\Pr\theta' + \Pr Nb\, \theta' S' + \Pr Nt \theta'^2 + 2Br(f')^2 + J f^2 = 0 \quad \ldots(8)$$

$$S'' - RLeS' + \frac{Nt}{Nb}\theta'' - K\, LeS = 0 \quad \ldots(9)$$

where $Pr = \frac{\mu C_P}{k_f}$ represents the Prandtl number and $Le = \frac{\upsilon}{D_B}$ represents the Lewis number, Grashof number is $Gr = \frac{(1-\phi) g \beta_T (T_2 - T_1) d^3}{\upsilon^2}$, $Re = \frac{u_0 d}{\upsilon}$ is the Reynold's number, $M^2 = \frac{B_0{}^2 d^2 \sigma}{\mu}$ is the magnetic parameter, $A = \frac{d^2}{\mu u_0}\frac{\partial p}{\partial x}$ is a constant pressure gradient, $Br = \frac{\mu u_0{}^2}{k_f (T_2 - T_1)}$ represents the Brinkman number, the Brownian motion parameter is $Nb = \frac{\tau D_B (\phi_2 - \phi_1)}{\upsilon}$, $Nt = \frac{\tau D_T (T_2 - T_1)}{T_m \upsilon}$ is the thermoporesis parameter and buoyancy ratio is $Nr = \dfrac{(\rho_p - \rho_{f_0})(\phi_2 - \phi_1)}{\rho_{f_0}\beta_T (T_2 - T_1)(1-\phi_m)}$, Joule heating parameter is $J = \dfrac{u_0{}^2 \sigma B_0^2 d^2}{(T_2 - T_1)k_f}$ and $K = \dfrac{k_1 d^2}{\upsilon}$.

The boundary conditions corresponding (5) are

$$\begin{aligned} &\theta = 0, \quad S = 0, \quad f = 0 \quad \text{at} \quad \eta = -1, \\ &\theta = 1, \quad S = 1, \quad f = 0 \quad \text{at} \quad \eta = 1. \end{aligned} \qquad \ldots(10)$$

## III. HOMOTOPY SOLUTION

In HAM to obtain the solution we take the initial values as (Liao, [14,15,16]),

$$f_0(\eta) = 0, \quad \theta_0(\eta) = \frac{\eta}{2} + \frac{1}{2}, \quad \text{and} \quad S_0(\eta) = \frac{\eta}{2} + \frac{1}{2}. \qquad \ldots(11)$$

And choose the auxiliary linear operators as$L_i = \partial^2/\partial y^2$, for$i = 1,2,3$.such that $L_1(c_1 + c_2 y) = 0$,$L_2(c_3 + c_4 y) = 0$ and $L_3(c_5 + c_6 y) = 0$, where $c_j$, $j = 1,2,3,4,5,6$.

The deformation of zero<sup>th</sup> order is given by

$$\begin{aligned} &L_1[f(\eta;p) - f_0(\eta)](1-p) = ph_1 N_1[f(\eta;p)], \\ &L_2[\theta(\eta;p) - \theta_0(\eta)](1-p) = ph_2 N_2[\theta(\eta;p)], \\ &L_3[S(\eta;p) - S_0(\eta)](1-p) = ph_3 N_3[S(\eta;p)]. \end{aligned} \qquad \ldots(12)$$

where

$$\begin{aligned} &N_1[f(\eta;p), \theta(\eta;p), S(\eta;p)] = f'' - Rf' + \frac{Gr}{Re}\left(\theta - Nr\,S\right) - M^2 f - A, \\ &N_2[f(\eta;p), \theta(\eta;p), S(\eta;p)] = \theta'' - R\Pr\theta' + \Pr Nb\theta' S' + \Pr Nt\theta'^2 + 2B_r (f')^2 + Jf^2, \\ &N_3[f(\eta;p), \theta(\eta;p), S(\eta;p)] = S'' - RLeS' + \frac{Nt}{Nb}\theta'' - K\,LeS. \end{aligned} \qquad \ldots(13)$$

Where embedded parameter is p$\in$ [0, 1] and auxiliary parameters are $h_i, (i = 1,2,3)$.

The boundary conditions are:

$$\theta(-1,p) = 0,\ S(-1,p) = 0,\ f(-1,p) = 0,\ \theta(1,p) = 1,\ S(1,p) = 1,\ f(1,p) = 0. \qquad \ldots(14)$$

Thus, f ,θ and S changes from $f_0$, $\theta_0$ and $S_0$ to the final solution f(η), θ(η) and S(η) as p=0 to p=1.
Using Taylor's series one can write

$$\begin{aligned} &f(\eta;p) = f_0 + \sum_{m=1}^{\infty} f_m(\eta) p^m, \quad f_m(\eta) = \frac{1}{m!}\frac{\partial^m f(\eta;p)}{\partial p^m}\bigg|_{p=0}, \\ &\theta(\eta;p) = \theta_0 + \sum_{m=1}^{\infty} \theta_m(\eta) p^m, \quad \theta_m(\eta) = \frac{1}{m!}\frac{\partial^m \theta(\eta;p)}{\partial p^m}\bigg|_{p=0}, \\ &S(\eta;p) = S_0 + \sum_{m=1}^{\infty} S_m(\eta) p^m, \quad S_m(\eta) = \frac{1}{m!}\frac{\partial^m S(\eta;p)}{\partial p^m}\bigg|_{p=0}. \end{aligned} \qquad \ldots(15)$$

And select the values of $h_i$ at p=1 where the series (15) are convergent i.e.

$$f(\eta) = f_0 + \sum_{m=1}^{\infty} f_m(\eta),\ \theta(\eta) = \theta_0 + \sum_{m=1}^{\infty} \theta_m(\eta),\ S(\eta) = S_0 + \sum_{m=1}^{\infty} S_m(\eta). \qquad \ldots(16)$$

The deformation of m<sup>th</sup> -order is given by

$$L_1[f_m(\eta)-\chi_m f_{m-1}(\eta)] = h_1 R_m^f(\eta),$$
$$L_2[\theta_m(\eta)-\chi_m \theta_{m-1}(\eta)] = h_2 R_m^\theta(\eta), \qquad \ldots (17)$$
$$L_3[S_m(\eta)-\chi_m S_{m-1}(\eta)] = h_3 R_m^S(\eta).$$

where

$$R_m^f(\eta) = f'' - Rf' + \frac{Gr}{Re}\left(\theta - NrS\right) - M^2 f - (1-\chi_m)A,$$

$$R_m^\theta(\eta) = \theta'' - RPr\theta' + PrNb\sum_{n=0}^{m-1}\theta'_{m-1-n}S'_n + PrNt\sum_{n=0}^{m-1}\theta'_{m-1-n}\theta'_n$$
$$+2Br\sum_{n=0}^{m-1} f'_{m-1-n} f'_n + J\sum_{n=0}^{m-1} f_{m-1-n} f_n, \qquad \ldots(18)$$

$$R_m^S(\eta) = S'' - RLeS' + \frac{Nt}{Nb}\theta'' - K\,LeS.$$

for an integer $m$

$$\chi_m = 1 \quad \text{for} \quad m > 1$$
$$= 0 \quad \text{for} \quad m \le 1 \qquad \ldots (19)$$

## IV. ENTROPY GENERATION

In vertical channel, the volumetric rate of local entropy generation of a nanofluid can be described as

$$S_G = \frac{K_f}{T_1^2}\left(\frac{\partial T}{\partial y}\right)^2 + \frac{\sigma B_0^2 u^2}{T_1} + \frac{2\mu}{T_1}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{RD}{\phi_1}\left(\frac{\partial \phi}{\partial y}\right)^2 + \frac{RD}{T_1}\left(\frac{\partial T}{\partial y}\right).\left(\frac{\partial \phi}{\partial y}\right) \qquad \ldots(20)$$

According to Bejan,[16], entropy generation number *Ns* is given by

$$Ns = \theta'^2 + \frac{1}{\Omega_1}\left(2B_r f'^2 + J f^2\right) + \phi_2 S'^2 + \phi_3\theta' S' \qquad \ldots(21)$$

The dimensionless coefficients are $\phi_2$ and $\phi_3$ given by

$$\phi_2 = \frac{RD}{K_f.\Omega_1}\left(\frac{\Omega}{\Omega_1}\right).\Delta\phi \;\; \phi_3 = \frac{RD}{K_f.\Omega_1}.\Delta\phi \qquad \ldots(22)$$

where $\Omega = \dfrac{\Delta\phi}{\phi_1}, \quad \Omega_1 = \dfrac{\Delta T}{T_1}$ and the characteristic rate of *Ns* is $\dfrac{K_f(\Delta T)^2}{d^2 T_1^2}$.

The equation (21) can be written as $Ns = Nv + Nh$ …(23)

Where the *Ns* due to heat transfer irreversibility is *Nh*, *Ns* due to viscous dissipation is *Nv*andBejan number *Be* is

$$Be = \frac{Ns}{Nh+Nv} \qquad \ldots (24)$$

## V. RESULTS AND DISCUSSION

The effect of chemical reaction *K*, Joule heating *J* and magnetic parameter *M* on entropy generation *Ns*, Bejan number *Be*, nanoparticle volume fraction, temperature and velocity are investigated and presented geometrically in figures 3 - 8. By taking *Le = 1, R = 1, Pr = 1, Tp = 0.1, A = 1, Nr = 1, Gr = 10, Re = 2, Nb =0.3.*

**Figure2(a-b)** represents the influence of the *K* on *Be* and *Ns*. **Figure 2(a)** shows that *Ns* enhanced with enhancement in the *K*. From **Figure 2(b)** it is clear that *Be* increases near the lower plate of the channel, meanwhile far away from the plate the trend is reversed and *Be* decreasing at upper plate of the channel as increase in *K*.

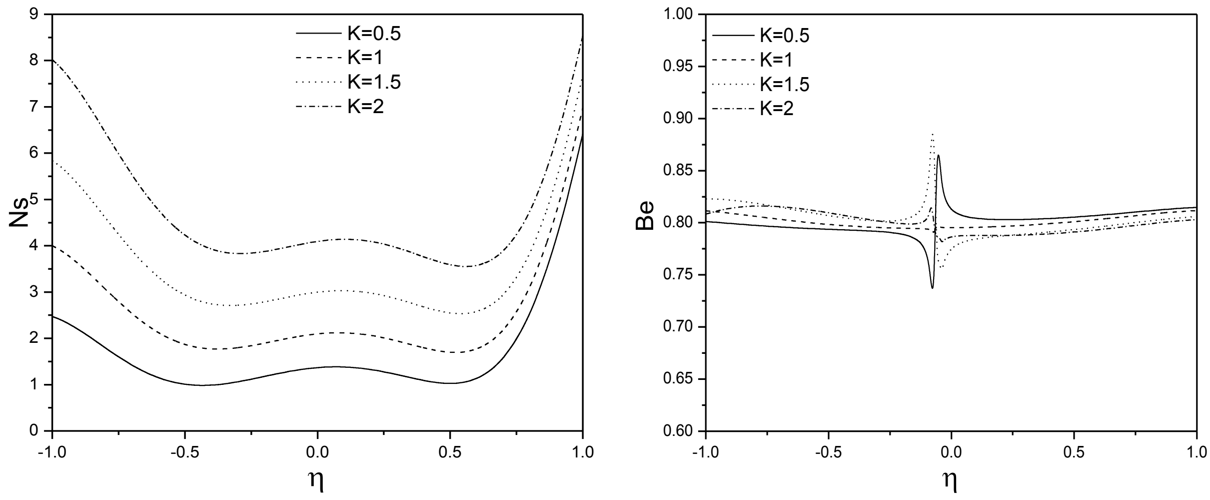
**Figure 3(a)**shows that the velocity increase as an enhance in the parameter *K*. This indicates that *K* have a retarding impact on the mixed convective flow. From **Figure 3(b)**, it is noticed that θ(η)increased with an increment in *K*. The impact of parameter *K* is to raises the temperature maximum in the flow field. From the flow region the heat energy released because of a raise in the chemical reaction therefore the fluid temperature increases. It is evident from **Figure 3(c)**that the S(η) decreases with a raise in *K*.

**Figure 4(a-b)** reveals the impact of the Joule heating parameter *J* on *Be* and *Ns*. **Figure 4(a)** shows that the enhance in *J* causes a increment in *Ns*. As increase in *J* the *Be* is observed as decreasing near the lower plate of the channel, meanwhile far away from the plate the trend is reversed and Be increasing near the upper plate of the channel as represented in **Figure 4(b).**

The influence of the Joule heating parameter *J* on the f(η), θ(η) and S(η) is shown in figure 5. The velocity f(η) raises with the rise in *J* as shown in **Figure 5(a).Figure 5(b)** show that the temperature enhanced with an increment in *J*. From **Figure 5(c)** it is identified that S(η) decreases with enhancement in *J*.

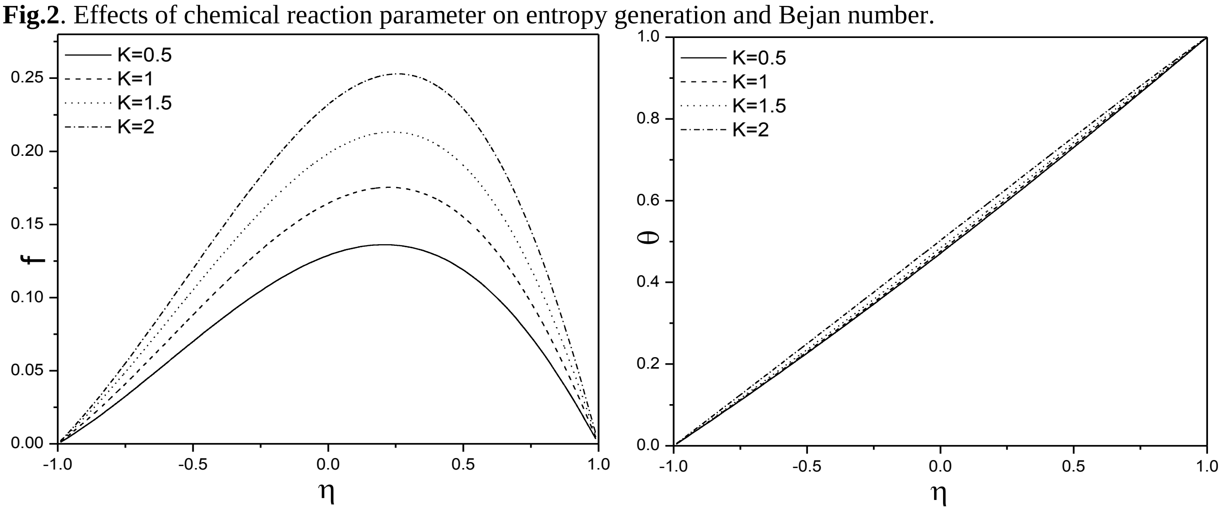
The influence of magnetic parameter *M* on *Be* and *Ns* is represented in **Figure 6(a-b)**. From **Figure 6(a)***Ns* decay with increment in *M*. From **Figure 6(b)** it is clear that *Be* increased near the lower plate of the channel, meanwhile far away from the plate the trend is reversed and *Be* decreasing at upper plate of the channel as enhance in the value of *M*.

The effects of the *Br on Be* and *Ns* is displays in **Figure 7(a-b)**. **Figure 7(a)**shows that an enhance in *Br* causes an increment in *Ns*. As increase in *Br* the Bejan number is observed as increasing near the end plate of the channel, while the trend is reversed at centre of the channel and *Be* increasing near the upper plate of the channel as represented in **Figure 7(b).**

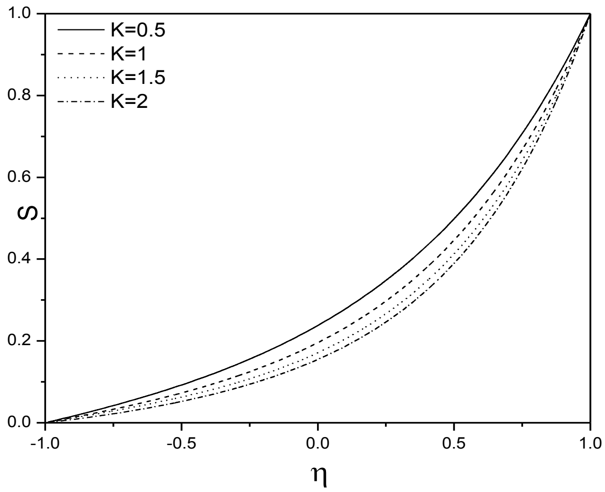



**(a)** Ns(η) **(b)** Be(η)

**Fig.2**. Effects of chemical reaction parameter on entropy generation and Bejan number.

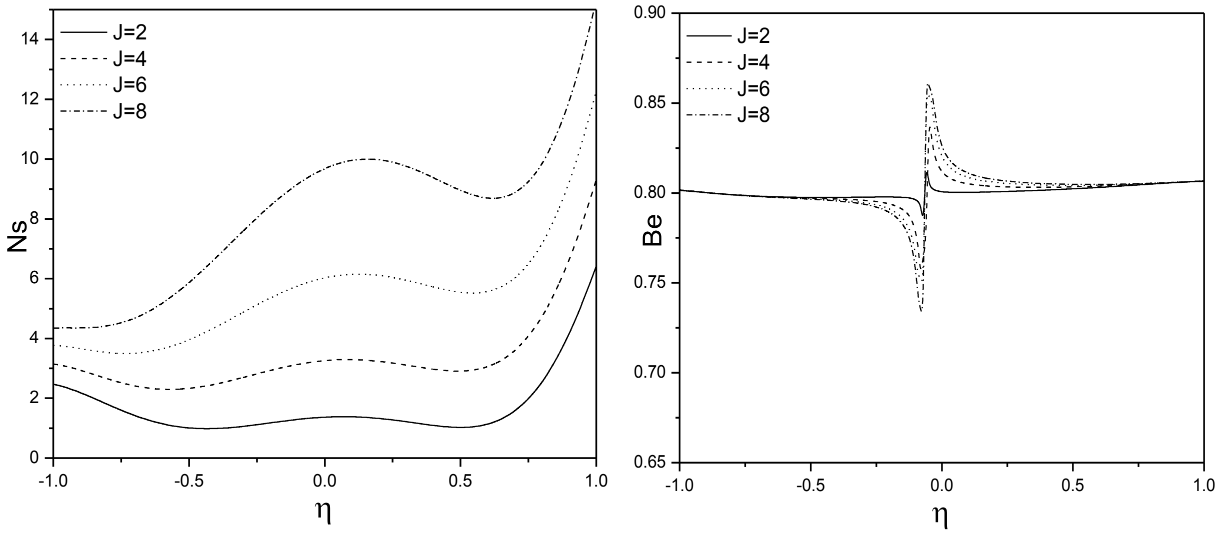



**(a)** f(η) **(b)** θ(η)



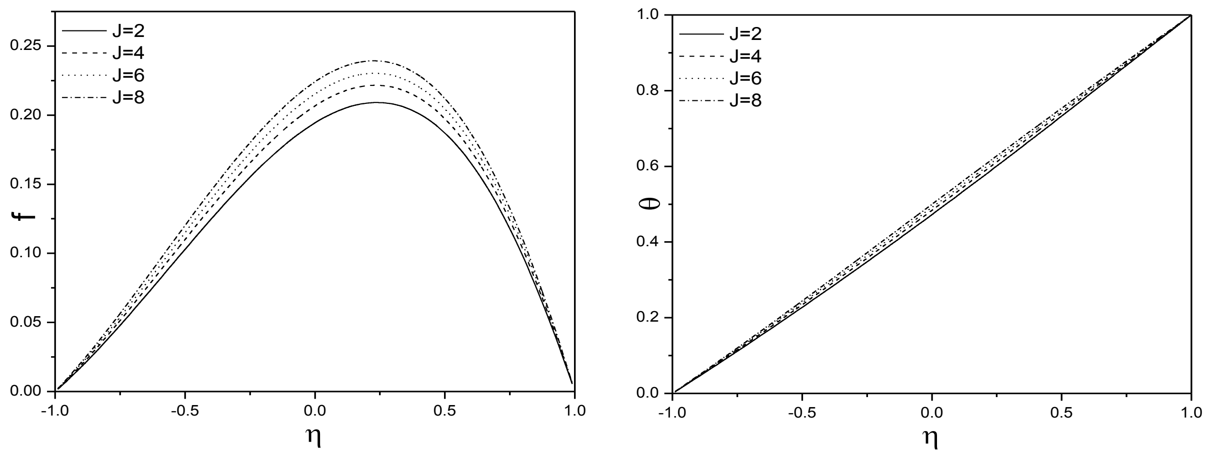

**(c)** S(η)

**Fig.3**. Effect *K* on velocity f(η), temperature θ(η) and nanoparticle concentration S(η).

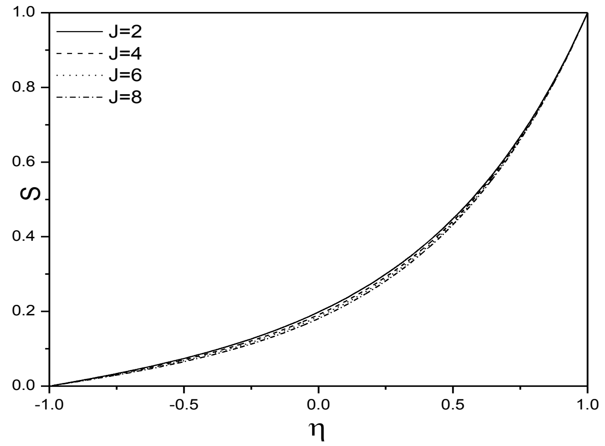



(**a**) Ns(η) (**b**) Be(η)

**Fig. 4**.Effect of Joule heating parameter on entropy generation and Bejan number.

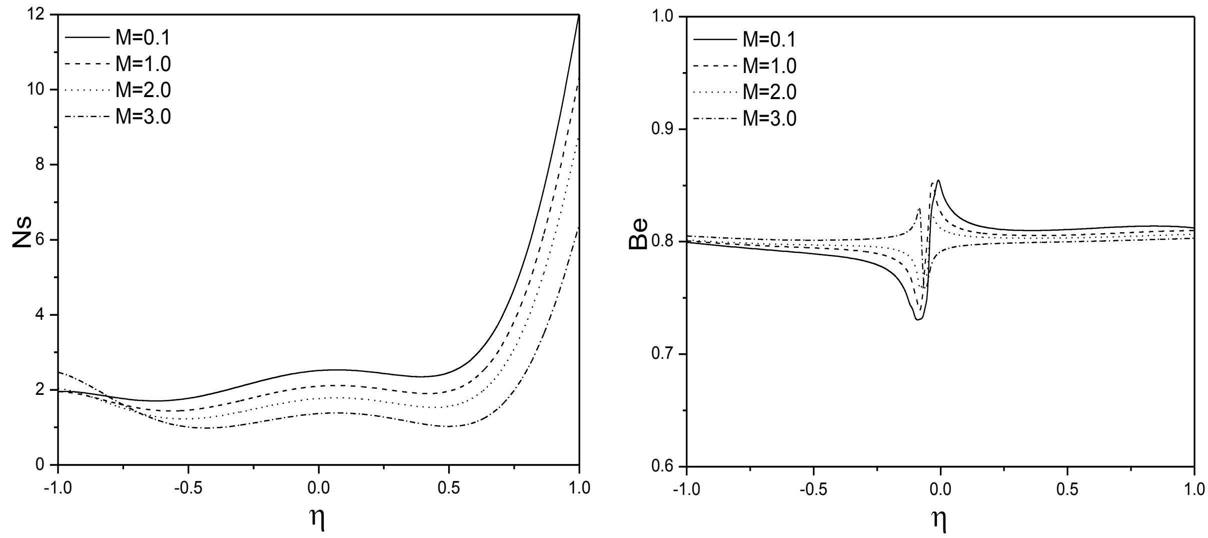



(**a**) f(η) (**b**) θ(η)



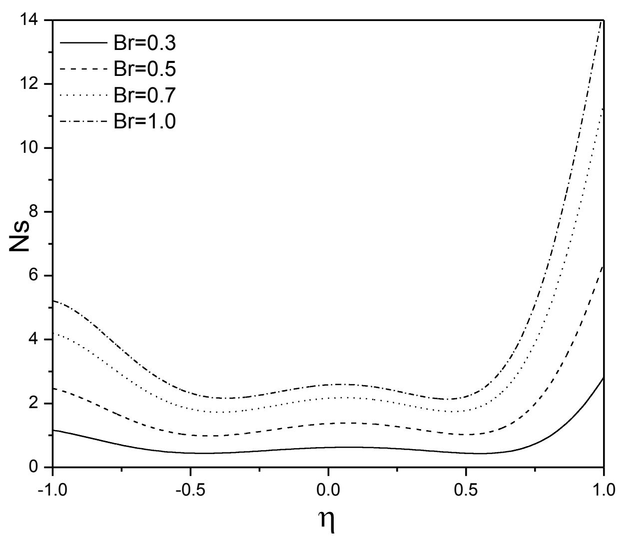

(**c**) S(η)

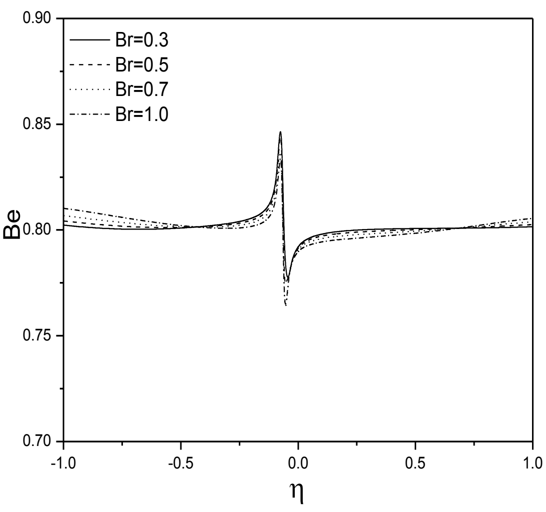
**Fig. 5**.Effect *J*on velocity f(η), temperature θ(η) and nanoparticle concentration S(η).




(**a**) Ns(η) (**b**) Be(η)

**Fig. 6**.Effect of magnetic parameter on entropy generation and Bejan number.




(**b**) Be(η)

(**a**) Ns(η)

**Fig. 7**.Effect of Brinkman number on entropy generation and Bejan number.

## VI. CONCLUSIONS

In this article entropy generation in mixed convective nanofluid flow in a vertical channel has been investigated. The maindecision are encapsulate below:

(a) The velocity, temperature and entropy generation increases whereas the nanoparticle concentration decreased with raise in chemical reaction *K*.

(b) The temperature, velocity and *Ns* increases but the nanoparticle volume fraction decreases as *J* increases.

(c) The extreme values of *Be* are identified at upper and lower plate of the channel by increasing the effects of parameters *Br, K, J,Nt* and *M*.

## REFERENCES

[1]. S.U.S. Choi, Enhancing thermal conductivity of fluids with nanoparticles. In Singer, D andWang, H (Eds.), Development and Applications of Non-Newtonian Flows, *American Society of Mechanical Engineers,*New York,231, 1995, 99–105.

[2]. X. Hang, F. Tao, I. Pop,Analysis of mixed convection flow of a nanofluid in a verticalchannel with the Buongiorno mathematical model, *International Communications in Heat Mass Transfer,* 44, 2013, 15-22.

[3]. D.A. Nield, A.V.Kuznetsov,Forced convection ina parallel -plate channel occupied by a nanofluid or a porous mediumsaturated by a nanofluid, *Int.J. Heat Mass Trans*, 70, 2014, 430-433.

[4]. S. Das, R.N. Jana, O.D. Makinde, Mixed convective magnetohydrodynamic flow in a vertical channel filled with nanofluids,*Engineering Science and Technology, an InternationalJournal*, 18, 2015, 244-255.

[5]. T. Hayat, Maryam Shafique, A. Tanveer, A. Alsaedi, Hall and ion slip effectson peristaltic flow of Jeffrey nano fluid with Joule heating,*Journal of Magnetism and Magnetic Materials*, 407, 2016, 51–59.

[6]. K. Takahashi and R. Alkire, Mass transferin Gas-Spargedporous electrodes,*Chemical Engineering Communications*, 38, 1985, 209-227.

[7]. H. Saleh, H. Ishak,S.Basriati,Flow Reversal of Fully Developed Mixed Convection in a Vertical Channel with ChemicalReaction, *International Journal of Chemical Engineering*, 2013.

[8]. M.Kothandapani, J.Prakash, Effects of thermal radiation and chemical reactions onperistaltic flow of a Newtonian nanofluid under inclined magnetic field ina generalized verticalchannel usinghomotopy perturbation method, *Asia-Pac.Journal of Chemical Engineering,*10, 2015, 259–272.

[9]. P. Dulal, T.Babulal, Combined effects of Joule heating and chemical reaction on unsteady MHD mixed convection of aviscous dissipating fluid over a verticalplate in porous media with thermal radiation, *Mathematical and Computer Modelling*, 54, 2011.

[10]. G. Mridulkumar,Effects of Chemical Reaction on MHD boundary layer flow over an exponentially stretching sheet withJoule Heating and Thermal Radiation, *International Research Journal of Engineering and Technology (IRJET),*2(9) , 2015.

[11]. A.Bejan, *Entropy generation through heat and fluid flow,*New York: Wiley; 1996.

[12]. A.Bejan, *Entropy generation minimization*, New York:Wiley; 1982.

[13]. J. Buongiorno, Convective transport in nanofluids, *ASME J Heat Transfer*, 128, 2006, 240 – 250.

[14]. S. J. Liao,On the homotopy analysismethod for nonlinear problems, *Appl.Math. Comp.*,147, 2004, 499–513.

[15]. S. J. Liao,An optimal homotopy-analysisapproach for strongly nonlinear differential equations,*Commun. Nonlinear Sci.Numer.Simul.*,15, 2010, 2003-16.

[16]. S. J. Liao,*Advances in the Homotopy Analysis Method,* World Scientific Publishing Company, Singapore, 2013.